

# One-Sample t-Test

## Example 1: Mortgage Process Time

### Problem

A faster loan processing time produces higher productivity and greater customer satisfaction. A financial services institution wants to establish a baseline for their process by estimating their mean processing time. They also want to determine if their mean time differs from a competitor's claim of 6 hours.

### Data collection

A financial analyst randomly selects 7 loan applications and manually calculates the time between loan initiation and when the customer receives the institution's decision.

### Tools

- **Descriptive Statistics**
- **1-Sample t**
- **Normality Test**
- **Time Series Plot**
- **Individual Value Plot**

### Data set

Mortgage.MPJ

Variable	Description
Loan	Loan application number
Hours	Number of hours until customer receives notification

## Displaying descriptive statistics

Use descriptive statistics to summarize important features of the data. In particular, descriptive statistics provide useful information about the location and variability of the data.

### Display Descriptive Statistics

1. Open Mortgage.MPJ.
2. Choose **Stat** > **Basic Statistics** > **Display Descriptive Statistics**.
3. In **Variables**, enter *Hours*.
4. Click **OK**.

## Interpreting your results

The statistics indicate that the sample mean is 5.079 hours. This is slightly below the target time of 6 hours. The one sample t-test compares this difference (0.921) to the variation in the data. While the standard deviation of the sample data is 1.319 hours, the one sample t-test uses the SE Mean.

### Descriptive Statistics: Hours

#### Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Hours	7	0	5.079	0.498	1.319	3.250	3.700	5.200	6.250	6.750

## Standard Error of the Mean

We can see that the SE Mean is simply the standard deviation divided by the square root of the number of data points and represents the dispersion or variation in the distribution of sample means. The one-sample t-test uses the distribution of the sample mean (not the distribution of the data) for the analysis. Therefore, the standard error of the mean will be used as the estimate of variation for the t-test and confidence interval.

### What is the standard error of the mean?

The standard error of the mean (SE of the mean) estimates the variability between sample means that you would obtain if you took multiple samples from the same population. The standard error of the mean estimates the variability between samples whereas the standard deviation measures the variability within a single sample.

For example, you have a mean delivery time of 3.80 days with a standard deviation of 1.43 days based on a random sample of 312 delivery times. These numbers yield a standard error of the mean of 0.08 days (1.43 divided by the square root of 312). Had you taken multiple random samples of the same size and from the same population the standard deviation of those different sample means would be around 0.08 days.

Use the standard error of the mean to determine how precisely the mean of the sample estimates the population mean. Lower values of the standard error of the mean indicate more precise estimates of the population mean. Usually, a larger standard deviation will result in a larger standard error of the mean and a less precise estimate. A larger sample size will result in a smaller standard error of the mean and a more precise estimate.

Minitab uses the standard error of the mean to calculate the confidence interval, which is a range of values likely to include the population mean.

# Hypothesis testing

## What is a hypothesis test

A hypothesis test uses sample data to test a hypothesis about the population from which the sample was taken. The one-sample t-test is one of many procedures available for hypothesis testing in Minitab.

For example, to test whether the mean duration of a transaction is equal to the desired target, measure the duration of a sample of transactions and use its sample mean to estimate the mean for all transactions. Using information from a sample to make a conclusion about a population is known as statistical inference.

## When to use a hypothesis test

Use a hypothesis test to make inferences about one or more populations when sample data are available.

## Why use a hypothesis test

Hypothesis testing can help answer questions such as:

- Are turn-around times meeting or exceeding customer expectations?
- Is the service at one branch better than the service at another?

For example,

- On average, is a call center meeting the target time to answer customer questions?
- Is the mean billing cycle time shorter at the branch with a new billing process?

# One-sample t-test

## What is a one-sample t-test

A one-sample t-test helps determine whether  $\mu$  (the population mean) is equal to a hypothesized value (the test mean).

The test uses the standard deviation of the sample to estimate  $\sigma$  (the population standard deviation). If the difference between the sample mean and the test mean is large relative to the variability of the sample mean, then  $\mu$  is unlikely to be equal to the test mean.

## When to use a one-sample t-test

Use a one-sample t-test when continuous data are available from a single random sample.

The test assumes the population is normally distributed. However, it is fairly robust to violations of this assumption for sample sizes equal to or greater than 30, provided the observations are collected randomly and the data are continuous, unimodal, and reasonably symmetric (see [1]).

## Why use a one-sample t-test

A one-sample t-test can help answer questions such as:

- Is the mean transaction time on target?
- Does customer service meet expectations?

For example,

- On average, is a call center meeting the target time to answer customer questions?
- Is the billing cycle time for a new process shorter than the current cycle time of 20 days?

## Testing the null hypothesis

The company wants to determine whether the mean time for the approval process is statistically different from the competitor's claim of 6 hours. In statistical terms, the process mean is the population mean, or  $\mu$  (mu).

### Statistical hypotheses

Either  $\mu$  is equal to 6 hours or it is not. You can state these alternatives with two hypotheses:

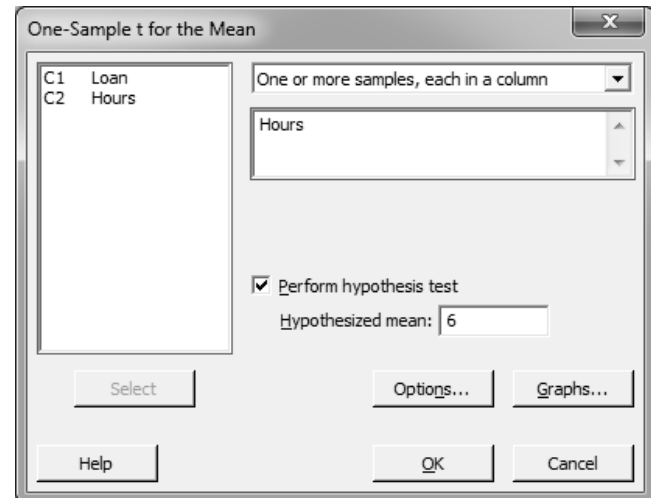
The *null hypothesis* ( $H_0$ ):  $\mu$  is equal to 6 hours.

The *alternative hypothesis* ( $H_1$ ):  $\mu$  is *not* equal to 6 hours.

Because the analysts will not measure every loan request in the population, they will not know the true value of  $\mu$ . However, an appropriate hypothesis test can help them make an informed decision. For these data, the appropriate test is a 1-sample t-test.

### 1-Sample t

1. Choose **Stat** > **Basic Statistics** > **1-Sample t**.
2. Complete the dialog box as shown below.



3. Click **OK**.

## Interpreting your results

### The logic of hypothesis testing

All hypothesis tests follow the same steps:

1. Assume  $H_0$  is true.
2. Determine how different the sample is from what you expected under the above assumption.
3. If the sample statistic is sufficiently unlikely under the assumption that  $H_0$  is true, then reject  $H_0$  in favor of  $H_1$ .

For example, the t-test results indicate that the sample mean is 5.079 hours. The test answers the question, "If  $\mu$  is equal to 6 hours, how likely is it to obtain a sample mean this different (or even more different)?" The answer is given as a probability value (P), which for this test is equal to 0.114.

### Test statistic

The t-statistic (-1.85) is calculated as:

$$t = (\text{sample mean} - \text{test mean}) / \text{SE Mean}$$

where SE Mean is the standard error of the mean (a measure of variability). As the absolute value of the t-statistic increases, the p-value becomes smaller.

## One-Sample T: Hours

### Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for $\mu$
7	5.079	1.319	0.498	(3.859, 6.298)

$\mu$  mean of Hours

### Test

Null hypothesis  $H_0: \mu = 6$   
 Alternative hypothesis  $H_1: \mu \neq 6$

T-Value	P-Value
-1.85	0.114



## Interpreting your results

### Making a decision

To make a decision, choose the significance level,  $\alpha$  (alpha), before the test:

- If P is less than or equal to  $\alpha$ , reject  $H_0$ .
- If P is greater than  $\alpha$ , fail to reject  $H_0$ . (Technically, you never *accept*  $H_0$ . You simply fail to reject it.)

A typical value for  $\alpha$  is 0.05, but you can choose higher or lower values depending on the sensitivity required for the test and the consequences of incorrectly rejecting the null hypothesis. Assuming an  $\alpha$ -level of 0.05 for the mortgage data, not enough evidence is available to reject  $H_0$  because P (0.114) is greater than  $\alpha$ .

### What's next

Check the assumption of normality.

## One-Sample T: Hours

### Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for $\mu$
7	5.079	1.319	0.498	(3.859, 6.298)

$\mu$ : mean of Hours

### Test

Null hypothesis  $H_0: \mu = 6$   
 Alternative hypothesis  $H_1: \mu \neq 6$

T-Value	P-Value
-1.85	0.114

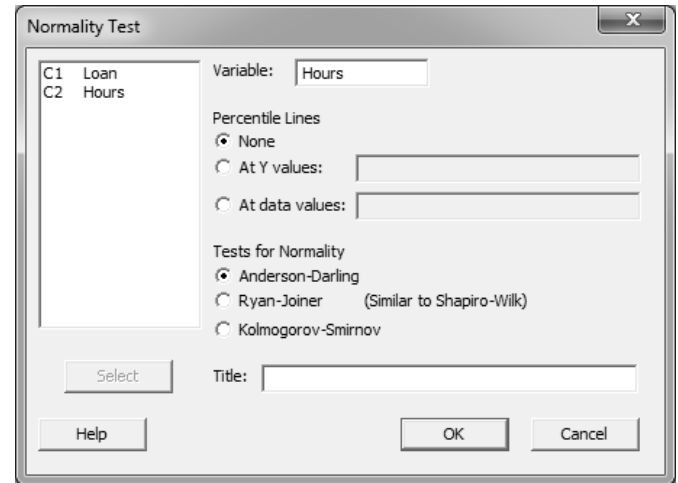
## Testing the assumption of normality

The 1-sample t-test assumes the data are sampled from a normally distributed population.

Use a normality test to determine whether the assumption of normality is valid for the data.

### Normality Test

1. Choose **Stat** > **Basic Statistics** > **Normality Test**.
2. Complete the dialog box as shown below.



3. Click **OK**.

## Interpreting your results

Use the normal probability plot to verify that the data do not deviate substantially from what is expected when sampling from a normal distribution.

- If the data come from a normal distribution, the points will roughly follow the fitted line.
- If the data do not come from a normal distribution, the points will not follow the line.

### Anderson-Darling normality test

The hypotheses for the Anderson-Darling normality test are:

$H_0$ : Data are from a normally distributed population

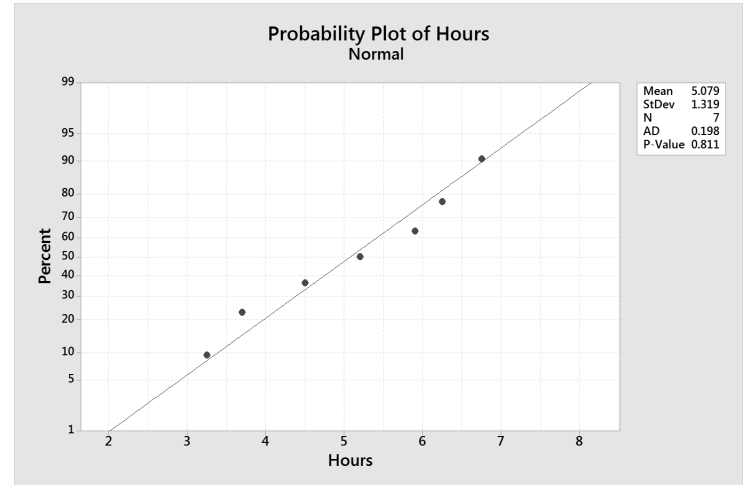
$H_1$ : Data are not from a normally distributed population

Using an  $\alpha$ -level of 0.05, there is insufficient evidence to suggest the data are not from a normally distributed population.

### Conclusion

Based on the plot and the normality test, assume that the data are from a normally distributed population.

**Note** When data are not normally distributed, you may be able to transform them using a Box-Cox transformation or use a nonparametric procedure such as the 1-sample sign test.



### What's next

Sort the data in time order to check the data for non-random patterns over time.

## Sorting data

You can use Minitab's Sort command to sort the data in ascending or descending order—numerically, alphabetically, or by date. In this example, sort the data by loan application number, in ascending, or increasing, order. Sorting the data by loan application number will put it in time order because the loan application numbers are assigned in increasing order by the system. Sorting the data chronologically makes it easy to plot the data over time and evaluate it for patterns or trends.

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**Note** Include all appropriate columns in the sorting step, to preserve the connection between the columns of data.

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### Sort

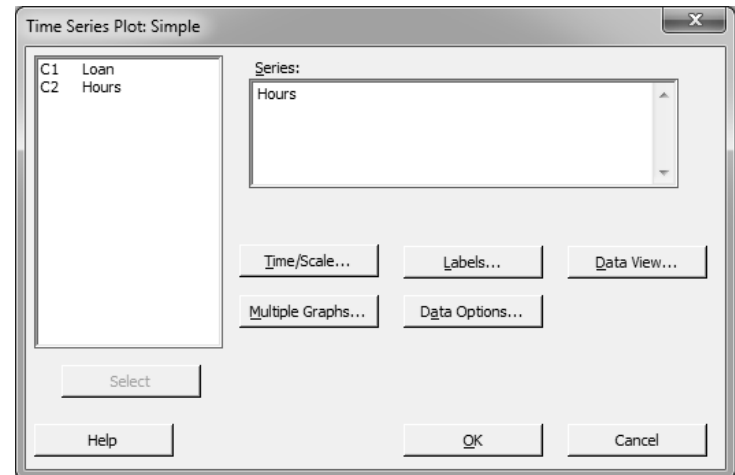
1. Choose **Data > Sort**.
2. Under **Column** in **Level 1**, enter *Loan*.
3. In **Storage location for the sorted columns**, select **In the original columns**.
4. Click **OK**.

## Testing the randomness assumption

Use a time series plot to look for trends or patterns in your data, which may indicate that your data are not random over time.

### Time Series Plot

1. Choose **Graph > Time Series Plot**.
2. Choose **Simple**, then click **OK**.
3. Complete the dialog box as shown below.



4. Click **OK**.

## Interpreting your results

If a trend or pattern exists in the data, we would want to understand the reasons for them. In this case, the data do not exhibit obvious trends or patterns.

### What's next

Calculate a confidence interval for the true population mean.



## Confidence intervals

### What is a confidence interval

A confidence interval is a range of likely values for a population parameter (such as  $\mu$ ) that is based on sample data. For example, with a 95% confidence interval for  $\mu$ , you can be 95% confident that the interval contains  $\mu$ . In other words, 95 out of 100 intervals will contain  $\mu$  upon repeated sampling.

### When to use a confidence interval

Use a confidence interval to make inferences about one or more populations from sample data, or to quantify the precision of your estimate of a population parameter, such as  $\mu$ .

### Why use a confidence interval

Confidence intervals can help answer many of the same questions as hypothesis testing:

- Is  $\mu$  on target?
- How much error exists in an estimate of  $\mu$ ?
- How low or high might  $\mu$  be?

For example,

- Is the mean transaction time longer than 30 seconds?
- What is the range of likely values for mean daily revenue?

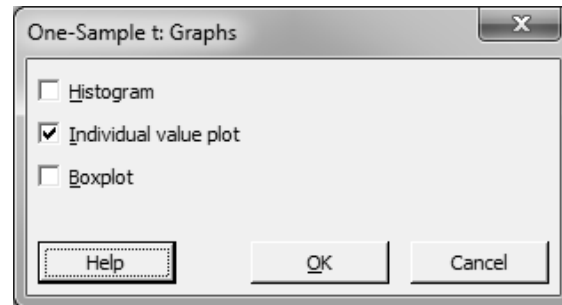
## Using the confidence interval

In the previous analysis, you used a hypothesis test to determine whether the mean of the mortgage processing time was different from the target value. You can also use a confidence interval to evaluate this difference.

The output pane results for 1-Sample t include values for the upper and lower bounds of the 95% confidence interval. Obtain a graphical representation of the interval by selecting **Individual value plot** in the **Graphs** subdialog box.

### 1-Sample t

1. Choose **Stat** > **Basic Statistics** > **1-Sample t**.
2. Click **Graphs**.
3. Complete the dialog box as shown below.



4. Click **OK** in each dialog box.



## Interpreting your results

### Confidence interval

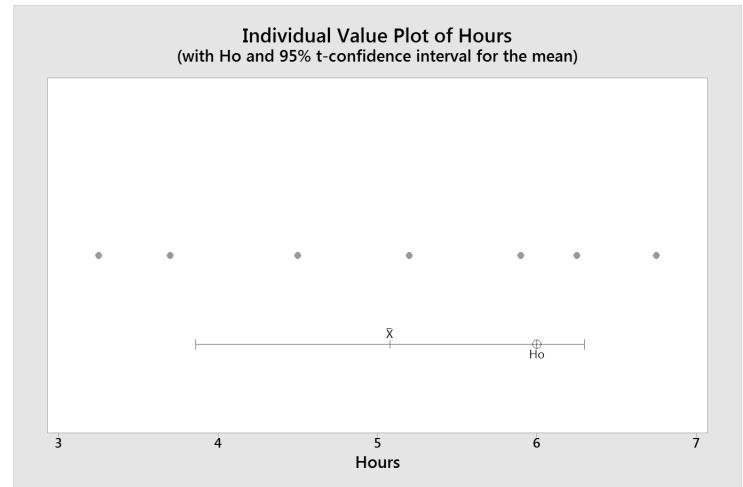
The confidence interval is a range of likely values for  $\mu$ . Minitab displays the interval graphically as a blue line on the individual value plot.

Under repeated sampling from the same population, the confidence intervals from about 95% of the samples would include  $\mu$ . Thus, for any one sample, you can be 95% confident that  $\mu$  is within the confidence interval.

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**Note** A confidence interval does not represent 95% of the data; this is a common misconception.

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## Interpreting your results

### Hypothesis test

The middle tick mark, labeled  $\bar{X}$ , represents the mean of the sample and the red circle, labeled  $H_0$ , represents the hypothesized population mean (6 hours). You can be 95% confident that the process mean is at least 3.859 hours, and at most 6.298 hours.

Use the confidence interval to test the null hypothesis:

- If  $H_0$  is outside the interval, the p-value for the hypothesis test will be less than 0.05. You can reject the null hypothesis at the 0.05  $\alpha$ -level.
- If  $H_0$  is inside the interval, the p-value will be greater than 0.05. You cannot reject the null hypothesis at the 0.05  $\alpha$ -level.

Because  $H_0$  falls within the confidence interval, you cannot reject the null hypothesis. Not enough evidence is available to conclude that  $\mu$  is different from the target of 6 hours at the 0.05 significance level.

### One-Sample T: Hours

#### Descriptive Statistics

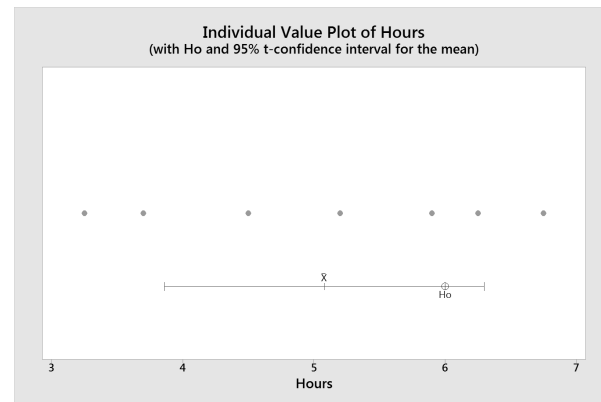
N	Mean	StDev	SE Mean	95% CI for $\mu$
7	5.079	1.319	0.498	(3.859, 6.298)

$\mu$ : mean of Hours

#### Test

Null hypothesis  $H_0: \mu = 6$   
 Alternative hypothesis  $H_1: \mu \neq 6$

T-Value	P-Value
-1.85	0.114



## Final considerations

### Summary and conclusions

According to the t-test and the sample data, you fail to reject the null hypothesis at the 0.05  $\alpha$ -level. In other words, the data do not provide sufficient evidence to conclude the mean processing time is significantly different from 6 hours.

The normality test and the time series plot indicate that the data meet the t-test's assumptions of normality and randomness.

The 95% confidence interval indicates the true value of the population mean is between 3.859 hours and 6.298 hours.

## Final considerations

### Hypotheses

A hypothesis test always starts with two opposing hypotheses.

The null hypothesis ( $H_0$ ):

- Usually states that some property of a population (such as the mean) is not different from a specified value or from a benchmark.
- Is assumed to be true until sufficient evidence indicates the contrary.
- Is never proven true; you simply fail to disprove it.

The alternative hypothesis ( $H_1$ ):

- States that the null hypothesis is wrong.
- Can also specify the direction of the difference.

### Significance level

Choose the  $\alpha$ -level *before* conducting the test.

- Increasing  $\alpha$  increases the chance of detecting a difference, but it also increases the chance of rejecting  $H_0$  when it is actually true (a Type I error).
- Decreasing  $\alpha$  decreases the chance of making a Type I error, but also decreases the chance of correctly detecting a difference.

### Assumptions

Each hypothesis test is based on one or more assumptions about the data being analyzed. If these assumptions are not met, the conclusions may not be correct.

The assumptions for a one-sample t-test are:

- The sample must be random.
- Sample data must be continuous.
- Sample data should be normally distributed (although this assumption is less critical when the sample size is 30 or more).

The t-test procedure is fairly robust to violations of the normality assumption, provided that observations are collected randomly and the data are continuous, unimodal, and reasonably symmetric (see [1]).

### Confidence interval

The confidence interval provides a likely range of values for  $\mu$  (or other population parameters).

You can conduct a two-tailed hypothesis test (alternative hypothesis of  $\neq$ ) using a confidence interval. For example, if the test value is not within a 95% confidence interval, you can reject  $H_0$  at the 0.05  $\alpha$ -level. Likewise, if you construct a 99% confidence interval and it does not include the test mean, you can reject  $H_0$  at the 0.01  $\alpha$ -level.