Using exponentially weighted moving average (EWMA) charts

Control charts are specialized time series plots, which assist in determining whether a process is in statistical control.

By Keith M. Bower

Some of the most widely-used form of control charts are $\overline{X}$-R charts and Individuals charts. These are frequently referred to as "Shewhart" charts after the control charting pioneer Walter Shewhart[1] who originated such techniques. These charts are sensitive to detecting relatively large shifts in the process, i.e. of the order of 1.5σ or above.

Two types of charts are primarily used to detect smaller shifts, namely Cumulative Sum (or CUSUM) charts and Exponentially Weighted Moving Average (EWMA) charts. E.S. Page[2] (1954) originally developed the CUSUM chart.

A CUSUM chart plots the cumulative sums of the deviations of each sample value from a target value. It has been used in various industries (especially the chemical industry) and the form of the CUSUM has been refined over the years to further increase its sensitivity (e.g. the Fast Initial Response, or FIR technique[3]).

Alternative technique

An alternative technique to detect small shifts is to use the EWMA methodology - developed by S.W. Roberts[4] in 1959. This type of chart has some very attractive properties, in particular:

1. Unlike $\overline{X}$-R and Individuals charts (without the Western Electric Handbook[5] rules which aim to increase sensitivity), all of the data collected over time may be used to determine the control status of a process.
2. The EWMA is often superior to the CUSUM charting technique for detecting "larger" shifts.
3. EWMA schemes may be applied for monitoring standard deviations in addition to the process mean.
4. There exists the ability to use EWMA schemes to forecast values of a process mean.
5. The EWMA methodology is not sensitive to normality assumptions.

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[1] Shewhart, W.A. (1931). *Economic Control of Quality of Manufactured Product*. Van Nostrand-Reinhold, New York.

[2] Page, E.S. (1954). "Continuous Inspection Schemes." *Biometrika*, Vol. 41, No. 1.

[3] Lucas, J.M., Crosier, R.B. (1982). "Fast Initial Response for CUSUM Quality Control Schemes." *Technometrics*, Vol. 24.

[4] Roberts, S.W. (1959). "Control Chart Tests Based on Geometric Moving Averages." *Technometrics*, Vol. 1.

[5] Western Electric (1956). *Statistical Quality Control Handbook*. Western Electric Corporation, Indianapolis, IN.

An important assumption that underpins the use of the EWMA (as well as other control charts) is that the samples obtained over time be independent. If that assumption is violated, there are two possible scenarios[6]:

a. Positive autocorrelation (e.g. low values tend to be followed by other low values, or high values tend to follow other high values). This can possibly lead to control limits that may be too narrow - positive correlation can increase the frequency of false alarms.
b. Negative autocorrelation (e.g. processes that frequently over-correct) may lead to overly wide control limits; hence special causes of variation that may be present in the process could be missed.

Consider the following simulated time series where the response variable is the concentration of an active chemical, expressed in grams per gallon.

Each individual batch takes several hours to be produced; hence analysis on the individual values may be appropriate. Suppose that we have knowledge of our process, namely that the historic mean is 20 grams/gallon and the within-subgroup standard deviation is 1 gram/gallon.

Underlying distribution

Using the first 30 observations to provide an estimate as to the possible underlying distribution, we note from Fig. 1 that the null hypothesis of normality cannot be rejected at the $\alpha = 0.10$ significance level, as the p-value of 0.420 associated with the Anderson-Darling test is greater than 0.10.

Note that Individuals control charts are more sensitive than $\overline{X}$ charts to the normality assumption. As Montgomery[7] states, "Even in situations where the normality assumption is violated to a slight or moderate degree… *[Shewhart]*…control charts will still work reasonably well." Though, as was noted by Schilling and Nelson[8], for subgroup sizes of less than 4, non-Normality can lead to serious problems (in particular, a high false alarm rate). Importantly, the EWMA structure is insensitive to Normality, whereas CUSUM charts are sensitive to Normal assumptions[9]. This makes the EWMA chart an attractive candidate in general when addressing small changes in a process.

Checking the autocorrelation function (ACF) results in Fig. 2, there does not appear to be a problem with the assumption of independence for the first 30 observations, as the ACF statistics fall inside the 95% confidence band.

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[6] Montgomery, D.C. (1996). *Introduction to Statistical Quality Control, 3rd Edition*. John Wiley & Sons.

[7] Montgomery, D.C. (1996). *Introduction to Statistical Quality Control, 3rd Edition*. John Wiley & Sons.

[8] Schilling, E.G., Nelson, P.R. (1976). "The Effect of Non-Normality on the Control Limits of $\overline{X}$ charts". *Journal of Quality Technology*, Vol. 25.

[9] Hawkins, D.M., Olwell, D.H. (1998). *Cumulative Sum Charts and Charting for Quality Improvement*, Springer-Verlag, New York.

As Fig. 3 shows, the Individuals chart (using the 3σ rule only) for the original 30 observations with 10 further observations after a 1σ shift has occurred indicates that the process is in statistical control. However, as indicated earlier, it should be noted that Individuals charts are not sensitive to small shifts in the process mean.

An EWMA chart may be used to detect small shifts. The parameters used are a constant (r) and some multiple (k) of the estimated value of σ. The EWMA for individual values may be defined as:

$Z_i = rX_i + (1 - r)Z_{i-1}$ where $0 < r \leq 1$, i =1, 2, …, n, n+1, …

Note that the average of some preliminary data ($\overline{X}$) is sometimes used for Z_0. Here we use $Z_0 = 20$ and the MINITAB default values of r = 0.2 and k = 3. Note that values of r = 1 and k = 3 are used in the regular Shewhart control charting methodology.

Small shifts

As the EWMA chart in Fig. 4 indicates, the process exhibits an out-of-control situation after the 38th data point. Importantly, this shift of 1σ, initiated after the 30th observation, was not detected using the Individuals control chart.

In conclusion therefore, one finds that EWMA charts are more sensitive than regularly used control charts to detect small shifts in a process. The non-necessity of Normally distributed data weighs in favor of the EWMA over the CUSUM, though as with all control charts, the assumption of independent subgrouping ought to be investigated.

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Figures

Fig 1

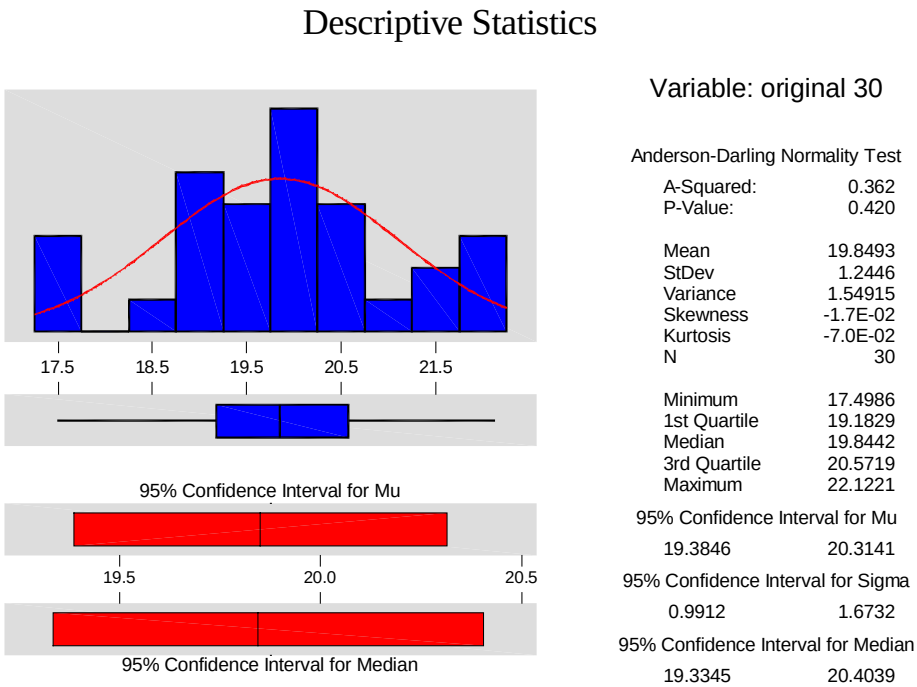
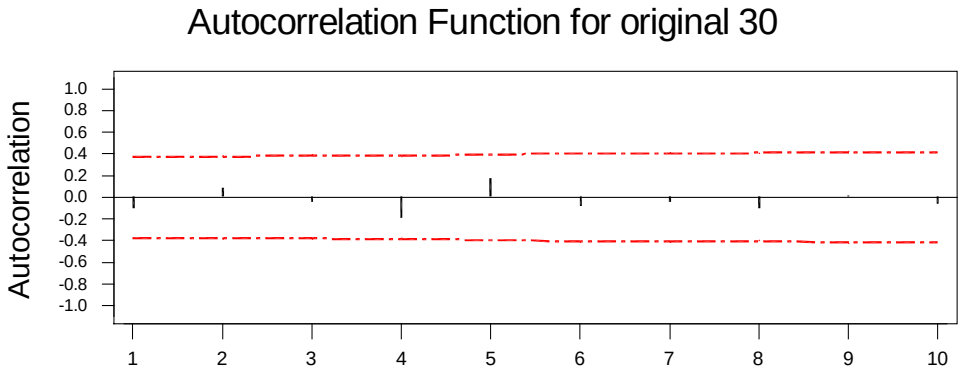


Fig 2



Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	-0.11	-0.59	0.39	8	-0.10	-0.50	4.08
2	0.09	0.46	0.64	9	0.02	0.07	4.09
3	-0.04	-0.23	0.71	10	-0.07	-0.34	4.31
4	-0.19	-1.03	2.08				
5	0.17	0.90	3.24				
6	-0.09	-0.44	3.54				
7	-0.05	-0.24	3.64				

Fig 3

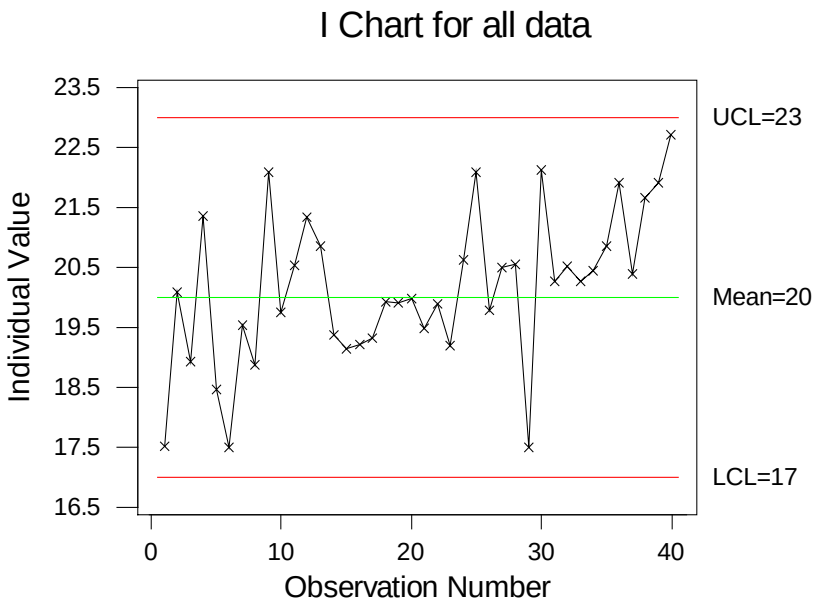
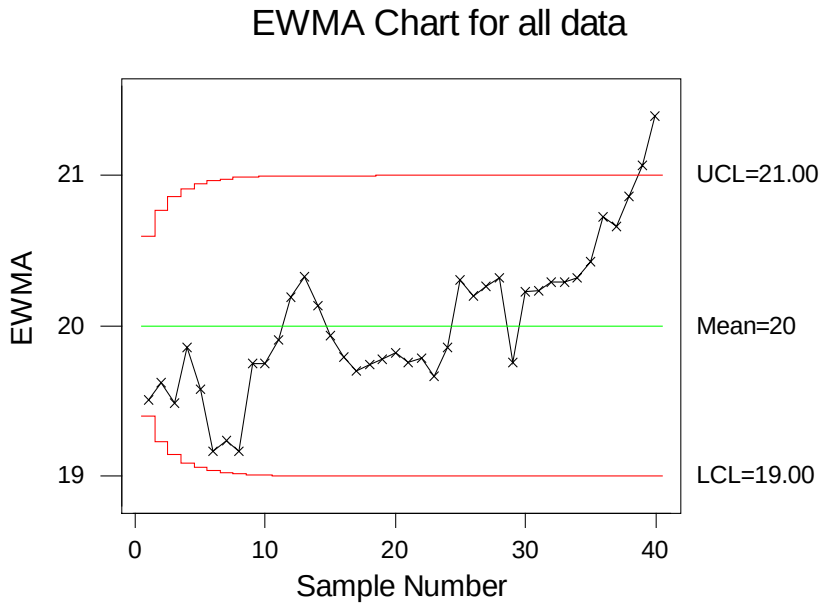

I Chart for all data
Individual Value
23.5
22.5
21.5
20.5
19.5
18.5
17.5
16.5
UCL=23
Mean=20
LCL=17
0
10
20
30
40
Observation Number

Fig 4


EWMA Chart for all data
EWMA
21
20
19
UCL=21.00
Mean=20
LCL=19.00
0
10
20
30
40
Sample Number