Using CUSUM Charts To Detect Small Process Shifts

By Keith M. Bower, M.S.

Abstract

In order to monitor a process, quality practitioners frequently use Shewhart control charts (e.g. $\overline{X}$–R, P-charts, etc.), so named after the pioneering work of Dr. Walter Shewhart[1]. It can be shown that if there are sharp, intermittent changes to a process, these types of charts are highly effective. However, if one is interested in a small, sustained shift in a process, other types of control charts may be preferred, for example the Cumulative Sum (CUSUM) chart, originally developed by Page[2] (1954). With exponential increases, an Exponentially Weighted Moving Average (EWMA) chart may be appropriate; for an example see Bower[3] (2000).

The CUSUM chart

Though CUSUM charts have been well researched and developed, it is true that many quality practitioners do not use them, even though there may be justifiable reasons to make use of this technique for their process. Possibly this may be due to a lack of instruction on CUSUM charts in many classes on SPC. In practice, however, I find that many of these same quality practitioners play the game of golf; hence they are in fact already well versed in the technique behind CUSUM charts.

In essence, for each hole in a round of golf, there are a specified number of times in which one should strike the ball, until it eventually drops into the hole. For example, on a par 4, if you strike the ball 4 times and it falls into the cup, then you held par. If you were able to do this task with only three shots (a “birdie”) then you are "1 under par" hence your cumulative sum is -1. This is continued throughout the course, the ultimate winner therefore having the lowest CUSUM.

Picture a golfer who is holding par for the first 13 holes, then suddenly hits form and has five successive birdies towards the end of the round. The final CUSUM is therefore –5, though from viewing a CUSUM chart it would be clear to see when the “process” shifted.

Example

Consider the following simulated manufacturing process involving a drill press, where we may reasonably estimate the process to be centered around 25 mm with a standard deviation of 2 mm. Currently, this process is being monitored by obtaining rational subgroups of size 5 at regular intervals, and that these selected parts are measured using an acceptable measuring system.

We shall firstly investigate this process using the traditional $\overline{X}$–R method. As is shown in Figure 1, the process appears to be in statistical control, however when looking at the associated CUSUM chart in Figure 2, one can see that the averages for each subgroup appear to increase over the final 10 subgroups. The increasing slope, representing the cumulative deviations from the target value of 25, exhibits this. Note that the target value choice is critical for a useful application of a CUSUM chart.

Figure 1:

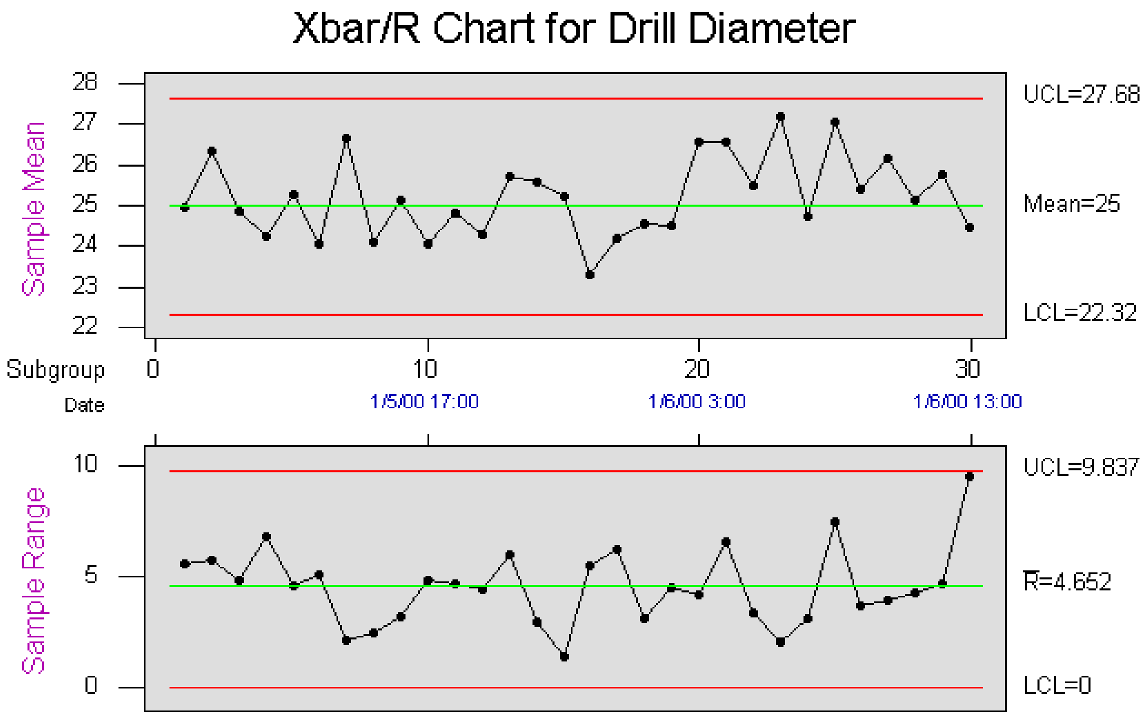
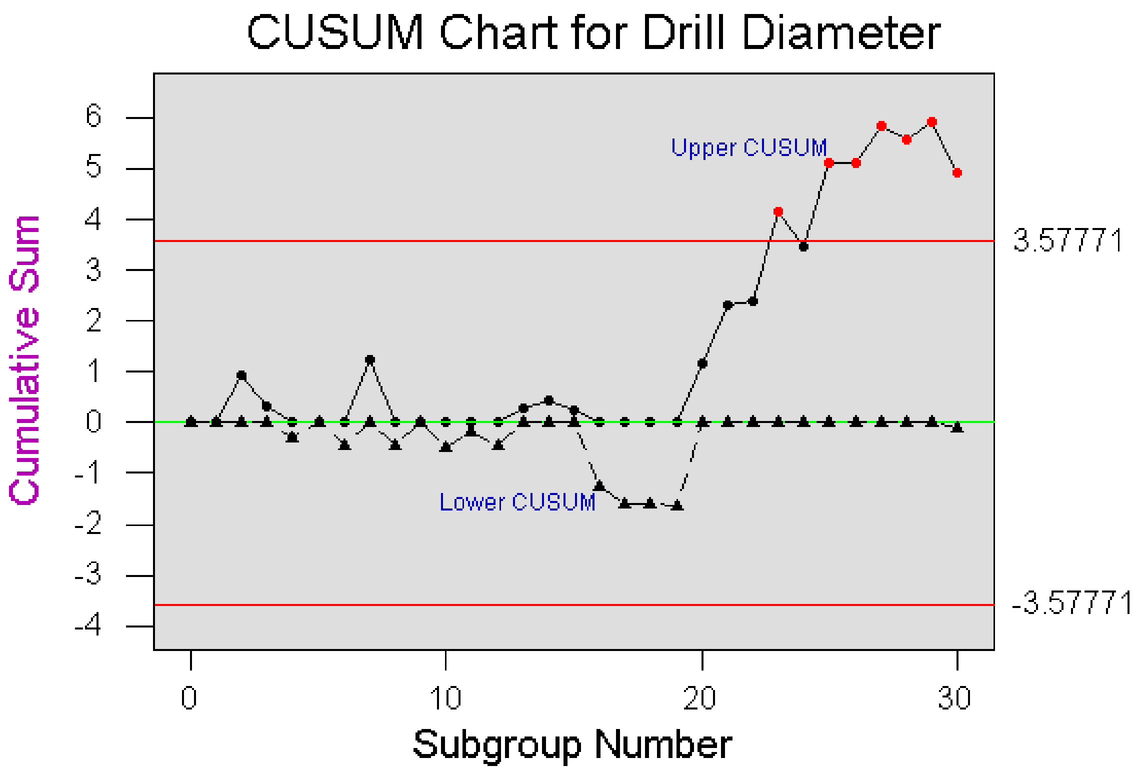


Figure 2:



Importantly, a small increase (of $\hat{\sigma}/2 = 1$ mm) was introduced for the final 10 subgroups. Consider, for instance, a small object becoming attached to the drill. This occurrence was clearly captured by the CUSUM chart in Figure 2, but not by the traditional $\overline{X}$–R chart in Figure 1.

It is possible to include additional rules to signal for an out of control situation, and the 2 out of 3 rule is violated for the 25th subgroup (unshown). It should be noted that increasing the number of tests performed would necessarily increase the false alarm rate. For more information see Champ and Woodall[4] (1987).

To obtain further information on CUSUM charting, including a discussion of upper and lower CUSUM scores, the interested reader is referred to Montgomery[5] (2000) and the references therein. For the purpose of this article, merely note the increase in the CUSUM around the 20th subgroup.

<u>Estimating Shift Sizes</u>

Another attractive feature of CUSUM charts is the ability to roughly estimate the size of the process shift. For example, one would note that from the 19th to the 30th subgroup, the CUSUM increases by roughly 0.9mm as shown in the results from the downloadable macro in Figure 3 (for more information on obtaining this macro, see reference 6). In fact, this is relatively close to the actual shift size of 1 mm.

Figure 3:

CUSUM shift results

```
length of period under study      11.0000
total CUSUM shift                 9.86000
average shift                     0.89636
within-subgroup sigma estimate    2.01671
% of estimated sigma              44.4468
```

In essence therefore, if one is interested in detecting a small and sustained shift in a process, a CUSUM chart is a useful vehicle to obtain such process knowledge. It is the hope of the author that the interested reader may seek further study in this interesting area of control chart methodology.

Keith M. Bower has an M.S. in Quality Management and Productivity from the University of Iowa, and is a Technical Training Specialist with Minitab, Inc.

Reference:

1. Shewhart, W.A., *Economic Control of Quality of Manufactured Product*, Van Nostrand-Reinhold, NY, (1931).
2. Page, E.S., Continuous Inspection Schemes, *Biometrika*, 41, (1954).
3. Bower, K.M., Using Exponentially Weighted Moving Average (EWMA) Charts, *Asia Pacific Process Engineer,* (October 2000). Also available from http://www.minitab.com/company/virtualpressroom/Articles/UsingEWMACharts.pdf
4. Champ, C.W. and Woodall, W.H., Exact Results for Shewhart Control Charts with Supplementary Runs Rules, *Technometrics*, 29, (1987).
5. Montgomery, D.C., *Introduction to Statistical Quality Control, 4th edition*, John Wiley & Sons (2000).
6. http://www.minitab.com/support/macros/index.asp?cat=QC_DOE#35