Monte Carlo Simulation with Minitab® Statistical Software

The Monte Carlo method is often used in Design for Six Sigma (DFSS) to analyze the sensitivity of a prototype system, and to predict yields and/or Cp and Cpk values. A product design should ideally have a small degree of sensitivity to process variation so its performance remains well within specification limits. If you use Minitab® Statistical Software, then you already have all the functionalities needed to run these types of studies.

THE MONTE CARLO APPROACH

The Monte Carlo method is a probabilistic technique based on generating a large number of random samples.

Simulations are particularly useful in the design phases of product development because they unravel the uncertainty or variability of a complex system.

An example from the semiconductor industry

This example shows how Monte Carlo methods may be implemented in a manufacturing process environment, to study the sensitivity of a process to variations.

In the semiconductor industry, Chemical Mechanical Polishing (CMP) processes are used to produce wafers that are as flat as possible to maintain high yield rates. A design of experiments is carried out to increase Removal Rates, leading to shorter cycle times. Six factors of the wafer manufacturing process are investigated: Down Force, Back Force, Oscillations, Type of PAD, Carrier Velocity, and Table Velocity.

The results from the Minitab DOE analysis show that, of the six factors, Down Force (DF), Carrier (CV) and Table (TV) Velocities, along with the Down Force* Carrier Velocity interaction (DF*CV), have a significant impact on the Removal Rate (RR).

The optimization tool in Minitab is used to identify the optimal settings to maximize the removal rate (shorter cycle times): Carrier Velocity needs to be kept low, whereas the levels of Down Force and Table Velocity have to be increased.

Optimal solutions are not enough, though. The robustness of the process window to manufacturing variations also needs to be taken into account. A sensitivity analysis based on a Monte Carlo simulation should be considered to estimate yields and Cpk values under standard operating conditions.

The Monte Carlo method may be divided into several steps:

- STEP 1

First, using the results of the DOE, identify the process inputs that have a statistically significant effect:

Removal Rate = 253 + 3.49 DF – 4.98 CV + 1.58 TV + 0.033 DF*CV

In order to conduct the Monte Carlo simulation, we need to identify the associated parametric distributions of the inputs. The objective is to generate a sample for each one of these variables from a distribution that has already been identified.

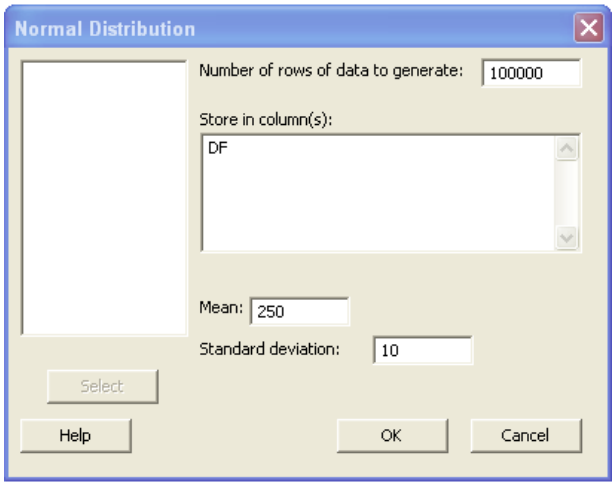
We expect the Down Force to follow a Normal distribution with a mean of 250 and a standard deviation of 10.

We expect the Carrier Velocity to follow a Normal distribution with a mean of 14.5 and a standard deviation of 3.

Finally, suppose that the Table Velocity follows a Weibull distribution with a shape of 4 and a scale of 11.

- STEP 2

Random samples are then generated for the inputs, according to their underlying distributions. Random sampling may be performed using Minitab's random data functionality. Go to **Calc** > **Random Data**, and select the desired distribution. Then specify the number of random variables you want to generate (100,000 in this example) and the supposed parameters for the distribution to store the results in columns in a Minitab worksheet:



- STEP 3

We then need to calculate numeric values for the output from the simulated inputs now stored in columns of the Minitab worksheet.

Use Minitab's calculator **Calc** > **Calculator** to introduce the transfer function, which is based on the final model from the DOE analysis:

Removal Rate = 253 + 3.49 DF – 4.98 CV + 1.58 TV + 0.033 DF*CV

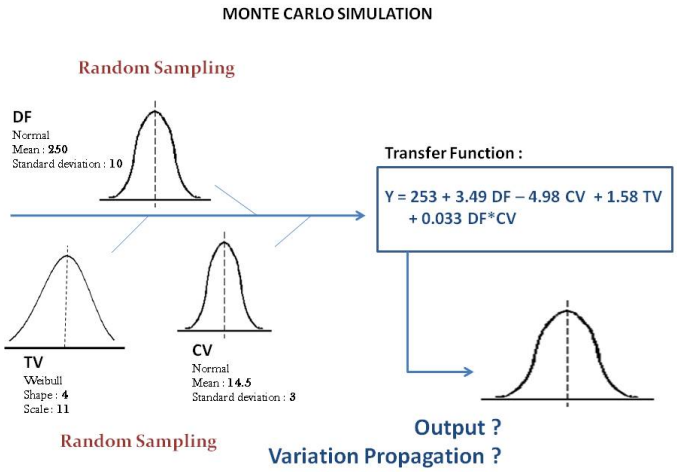


Diagram 1. Example of a Monte Carlo Simulation

It is now possible to analyze the distribution of the simulated output variable. A variety of statistical and graphical tools are available within Minitab to help you study the distribution of the output according to your objectives.

For example, you can perform a process capability analysis to assess the expected behavior of the output (**Stat** > **Quality Tools** > **Capability Analysis**). This capability analysis provides a better understanding of how much variability in the output can be expected in normal operating conditions.

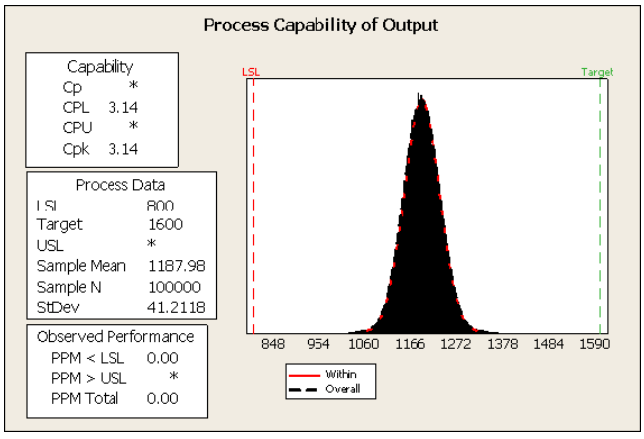


Diagram 2. Example of an analysis of the simulated output: a Capability Analysis

Whenever the output remains too sensitive, and no satisfactory solution can be found to limit the effects of input variations, process engineers need to focus their attention on controlling the process inputs that are directly linked to the output.

CONCLUSION

Minitab's ability to quickly generate a very large number of simulated values, along with its integrated statistical and graphic capabilities, make it a very powerful tool for Monte Carlo simulation methods.

For more information about performing Monte Carlo simulations in Minitab, including a collection of Minitab macros, visit:

www.minitab.com/support/answers/answer.aspx?id=2219

Bruno Scibilia
Technical Training Specialist, Minitab Sarl