

Analysis of Variance (ANOVA) Using Minitab

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Frequently, scientists are concerned with detecting differences in means (averages) between various levels of a factor, or between different groups. What follows is an example of the ANOVA (Analysis of Variance) procedure using the popular statistical software package, Minitab.

ANOVA was developed by the English statistician, R.A. Fisher (1890-1962). Though initially dealing with agricultural data[1], this methodology has been applied to a vast array of other fields for data analysis. Despite its widespread use, some practitioners fail to recognize the need to check the validity of several key assumptions before applying an ANOVA to their data. It is the hope that this article may provide certain useful guidelines for performing basic analysis using such a software package.

For this example we shall consider a set of data from the Journal of the Electrochemical Society [1992][2]. This data originated from an experiment performed to investigate the low-pressure vapor deposition of polysilicon. Four wafer positions have been chosen and our goal is to detect whether there are any statistically significant differences between the means of these levels.

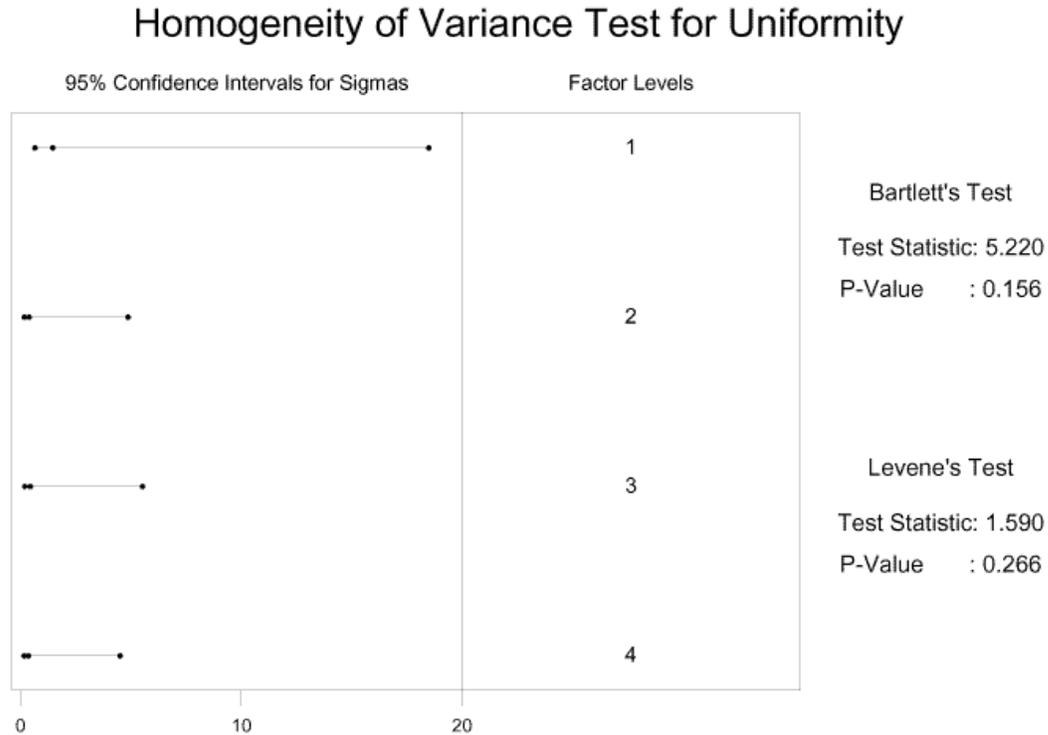
As is discussed by Hogg and Ledolter [1987][3], assumptions that underpin the ANOVA procedure are:

- (1) The values for each level follow a Normal (a.k.a. Gaussian) distribution, and
- (2) The variances are the same for each level (Homogeneity of Variance).

Traditionally, Normality has been investigated using Normal probability plots. However as is discussed by Ryan and Joiner [1976][4] inexperienced practitioners have difficulty in their interpretation, and considerable practice is sometimes necessary. With the advent of computer technology, statistical tests may be easily performed to investigate an assumed distributional form, e.g. the Anderson-Darling test. As this dataset is so small (only three observations for each level) it would be unusual to reject the null hypothesis of Normality, though an example of checking Normality will follow later.

With regard to assumption (2), namely homogeneous variances, as the output in Fig. 1 shows, we are unable to reject the null hypothesis of equal variances at the 0.05 significance level as the p-value for Bartlett's test (which assumes Normality within each factor level) is greater than 0.05. Levene's test does not assume Normality and also fails to reject the null hypothesis of equal variances.

Figure 1.



We may therefore proceed with our analysis. One finds that the ANOVA procedure works quite well even if the Normality assumption has been violated, unless one or more of the distributions are highly skewed or if the variances are quite different. Transformations of the original dataset may correct these violations, but are outside the scope of this article (see Montgomery [1997][5] for further information).

As the ANOVA table in Fig. 2 shows, we obtain an F-statistic of 8.29. Traditionally, one would seek to compare such a statistic with critical values from a table. With more sophisticated software packages, however, the p-value is automatically computed. In this example we are able to reject the null hypothesis, even at the 0.01 significance level. The p-value tells us that the lowest significance level attainable is only 0.008. This indicates that the wafer position level accounts for a significant amount of variation in the response variable of Uniformity. Therefore, there is very strong evidence to suggest that the means are not all equal. It is pertinent now to discover which levels have significantly different means.

Figure 2

General Linear Model: Uniformity versus Position

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Factor      Type      Levels  Values
Position    fixed      4       1 2 3 4

Analysis of Variance for Uniformity, using Adjusted SS for Tests

Source      DF      Seq SS      Adj SS      Adj MS      F      P
Position    3       16.2198     16.2198     5.4066     8.29  0.008
Error       8       5.2175     5.2175     0.6522
Total      11      21.4373

Unusual Observations for Uniformity

Obs  Uniformi      Fit      SE Fit  Residual  St Resid
  1   2.76000    4.30667    0.46626  -1.54667  -2.35R
  2   5.67000    4.30667    0.46626   1.36333   2.07R

R denotes an observation with a large standardized residual.

Tukey Simultaneous Tests
Response Variable Uniformity
All Pairwise Comparisons among Levels of Position

Position = 1 subtracted from:

Level      Difference      SE of      Adjusted
Position    of Means  Difference  T-Value    P-Value
  2          -2.533      0.6594     -3.842     0.0206
  3          -2.380      0.6594     -3.609     0.0283
  4          -2.990      0.6594     -4.535     0.0083

Position = 2 subtracted from:

Level      Difference      SE of      Adjusted
Position    of Means  Difference  T-Value    P-Value
  3           0.1533    0.6594     0.2325     0.9952
  4          -0.4567    0.6594    -0.6926     0.8971

Position = 3 subtracted from:

Level      Difference      SE of      Adjusted
Position    of Means  Difference  T-Value    P-Value
  4          -0.6100    0.6594    -0.9251     0.7928

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The results from Tukeys simultaneous tests in Fig. 2 indicate that the mean level for wafer position 1 is significantly higher than that of the other wafer positions (the corresponding p-values being less than 0.05). However, wafer positions 2, 3 and 4 are not significantly different from each other.

Note that we may consider a model of the form $Uniformity_{ij} = \mu + \tau_i + \varepsilon_{ij}$ where $i = 1, 2, 3, 4$; $j = 1, 2, 3$. The final and necessary step is to check the errors (the ε_{ij} 's) in the model using residual diagnostic tools. We assume that the errors:

- (1) Exhibit constant variance,
- (2) Are Normally distributed,
- (3) Have a mean of zero,
- (4) Are independent from each other.

When these assumptions hold, the ANOVA is an exact test of the null hypothesis of no difference in level means and we need to check these assumptions using the residuals. The residuals are the actual values minus the fitted values from the model. Minitab provides the fitted values and the residuals and we may assess these assumptions as follows.

From Fig. 3 we see that the plot of residuals vs. fitted values has more spread in the points for the highest fitted values (corresponding to level 1), though with so few points it is difficult to reject assumption (1) of constant variance in the residuals. The residuals appear to have somewhat of a bell-shaped curve, though the Normal probability plot has two values off the straight line at either end (corresponding to high and low values in level 1). However, as is shown in Fig. 4 we see directly that assumptions (2) and (3) appear to be met as the p-value for the Anderson Darling test for Normality is greater than 0.05. Hence we are unable to reject the null hypothesis of Normality, and also that the mean of the residuals is zero. The I-chart (Individuals control chart) in the top right hand corner of Fig. 3 assesses the independence assumption, and does not exhibit any concerning features.

Figure 3

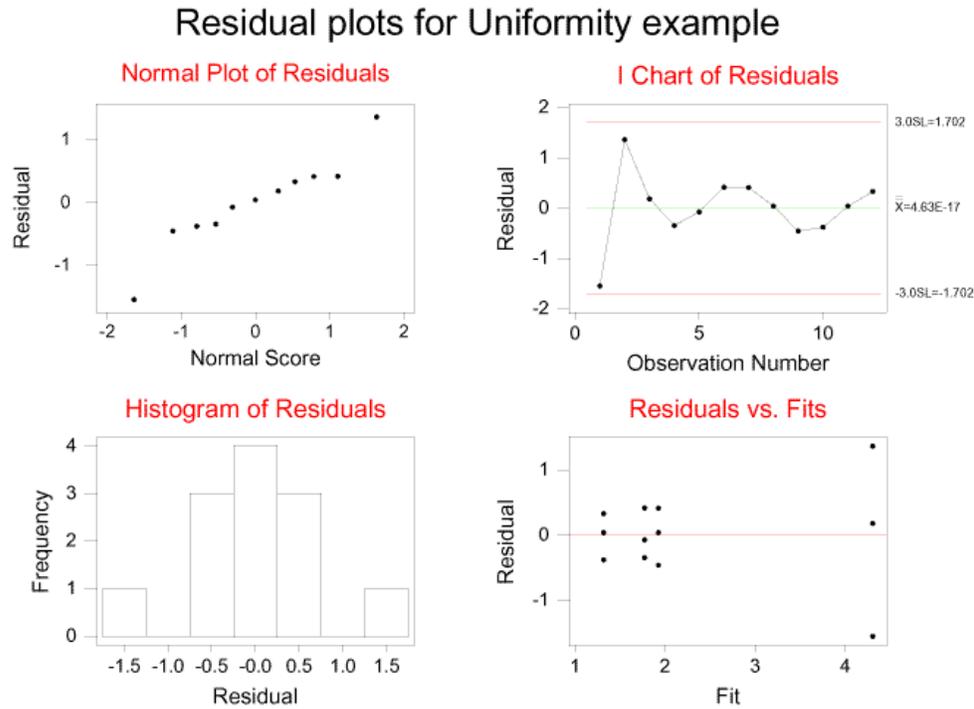
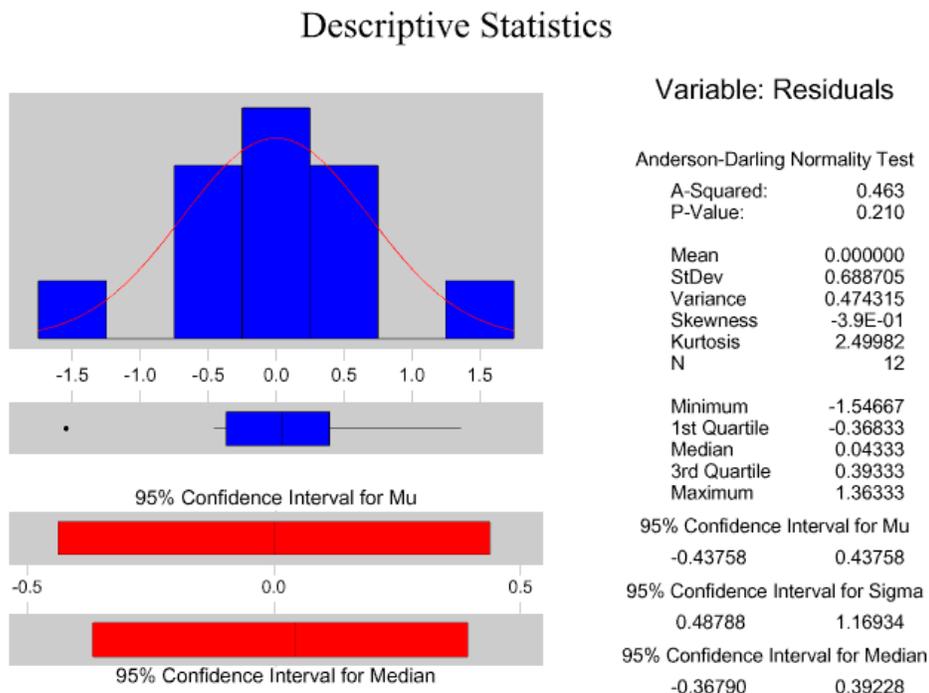


Figure 4



When appropriate, the use of the correct statistical technique can lead to very useful answers. The assumptions for the statistical techniques, although often overlooked, are crucial to the successful interpretation of results. With the aid of advanced statistical software packages results are quickly, easily and reliably obtained.

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[3] Hogg, R.V., Ledolter, J. (1987). Applied Statistics for Engineers and Physical Scientists. Macmillan Publishing Company, NY.

[4] Ryan, T.A., Joiner, B.L. (1976). Normal Probability Plots and Tests for Normality.

[5] Montgomery, D.C. (1997). Design and Analysis of Experiments, 4th Edition. John Wiley & Sons, Inc.

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