

## Technical Support Document

# Formulas for Estimating the Standard Deviation for Capability Analysis and Sixpack (Between/Within) and I-MR-R/S (Between/Within) Charts

### $\sigma_{\text{Within}}$

$\sigma_{\text{Within}}$  is the standard deviation for the variation within a batch (used for the range or S chart). This sigma can be estimated based on a pooled standard deviation (the default), the average of the subgroup ranges ( $\bar{r}$ ), or the average of the subgroup standard deviations ( $\bar{s}$ ). See “Methods of Estimating Standard Deviation” below. Note: This will match the within standard (StDev (Within)) reported by Capability Analysis (Normal).

### $\sigma_{\text{Between}}$

$\sigma_{\text{Between}}$  is the standard deviation for the variation from batch to batch:

$\sigma_{\text{Between}} = \text{The square root of } (\max\{0, \sigma_{\bar{X}}^2 - (\sigma_{\text{Within}}^2 / \text{batch size})\})$   
where batch size is the mode of the subgroup sizes

$\sigma_{\bar{X}}$  is the standard deviation of the batch sample means (used for the individuals chart and moving range chart of means). This sigma can be estimated based on the average moving range (the default), the median moving range or the square root of the MSSD. See “Methods of Estimating Standard Deviation” below.

### $\sigma_{\text{B/W}}$

$\sigma_{\text{B/W}}$  is the total standard deviation incorporating variation within and between batches (used for between/within capability statistics).

$\sigma_{\text{B/W}} = \text{The square root of } (\sigma_{\text{Between}}^2 + \sigma_{\text{Within}}^2)$

### $\sigma_{\text{Overall}}$

$\sigma_{\text{Overall}}$  is the overall standard deviation. See “Methods of Estimating Standard Deviation” below.

Note: This will match the overall standard (StDev (Overall)) reported by Capability Analysis (Normal).

You can estimate the standard deviation either using unbiasing constants or not. By default, Minitab 15 uses unbiasing constants when estimating the within and between subgroup standard deviations, and by default does *not* use unbiasing constants when estimating the overall standard deviation. By default, Minitab 14 uses unbiasing constants when estimating the within and between subgroup standard deviations, and always uses unbiasing constants when estimating the overall standard deviation. To change the default(s) for the current session, choose Stat > Quality Tools > Capability Analysis > Between/Within > Estimate. To change the default(s) for future sessions, choose Tools > Options > Control Charts and Quality Tools > Estimating Standard Deviation.

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### Methods of Estimating Standard Deviation

#### Estimating $\sigma_{\text{Within}}$

##### Pooled standard deviation - Default

$$\hat{\sigma}_{\text{within}} = \frac{s_p}{c_4(d)}$$

$$s_p = \sqrt{\frac{\sum_i \sum_j (x_{ij} - \bar{x}_i)^2}{\sum_i (n_i - 1)}}$$

$d = 1 + \sum (n_i - 1)$  degrees of freedom

##### Rbar – Average of subgroup ranges

$$\hat{\sigma}_{\text{within}} = \frac{\sum_i \left( \frac{f_i r_i}{d_2(n_i)} \right)}{\sum_i f_i}$$

$$f_i = \frac{(d_2(n))^2}{(d_3(n))^2}$$

$$\hat{\sigma}_{\text{within}} = \frac{\bar{R}}{d_2(n_i)} \text{ if all } n_i \text{ are the same}$$

##### Sbar - Average of subgroup standard deviation

$$\hat{\sigma}_{\text{within}} = \frac{\sum_i \left( \frac{h_i s_i}{c_4(n_i)} \right)}{\sum_i h_i}$$

$$s_i = \sqrt{\frac{\sum_j (x_{ij} - \bar{x}_i)^2}{n_i - 1}}$$

$$h_i = \frac{(c_4(n_i))^2}{1 - (c_4(n_i))^2}$$

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### Estimating $\sigma_{XBar}$ (to calculate $\sigma_{Between}$ )

#### Average moving range - Default

$$\hat{\sigma}_{XBar} = \frac{MR_{\bar{x}}}{d_2(w)}$$

$MR_{\bar{x},i}$  = the  $i^{\text{th}}$  moving range of the subgroup means =  $\max[\bar{x}_i, \dots, \bar{x}_{i-w+1}] - \min[\bar{x}_i, \dots, \bar{x}_{i-w+1}]$ ,  
for  $i = w, \dots, n$

$$\overline{MR}_{\bar{x}} = \frac{MR_{\bar{x},w} + \dots + MR_{\bar{x},n}}{(n-w+1)}$$

$w$  = the number of observation used in the moving range.  $w=2$  by default.

#### Median moving range

$$\hat{\sigma}_{XBar} = \frac{\widetilde{MR}_{\bar{x}}}{d_4(w)}$$

$MR_{\bar{x},i}$  = the  $i^{\text{th}}$  moving range of the subgroup means =  $\max[\bar{x}_i, \dots, \bar{x}_{i-w+1}] - \min[\bar{x}_i, \dots, \bar{x}_{i-w+1}]$ ,  
for  $i = w, \dots, n$

$\widetilde{MR}_{\bar{x}}$  = the median of all  $MR_{\bar{x},i}$

$w$  = the number of observations used in the moving range.  $w=2$  by default.

### Square root of MSSD – Square root of the Mean of the Squared Successive Differences

$$\hat{\sigma}_{XBar} = \frac{\sqrt{\frac{\sum d_i^2}{2(n-1)}}}{c_4(n_i)}$$

$d_i$  = successive differences

$$c_4(n_i) \cong c_4(n_i)$$

### When you specify a historical value for $\sigma_{Between}$

If you provide a historical value for  $\sigma_{Between}$  Minitab uses the following formula to calculate  $\sigma_{Xbar}$

$\sigma_{Xbar}$  = The square root of (  $\sigma_{Between}^2 + (\sigma_{Within}^2 / \text{batch size})$  )

where batch size is the mode of the subgroup sizes

### Estimating overall standard deviation

$$\hat{\sigma}_{overall} = S = \sqrt{\frac{\sum_i \sum_j (x_{ij} - \bar{x})^2}{n-1}}$$